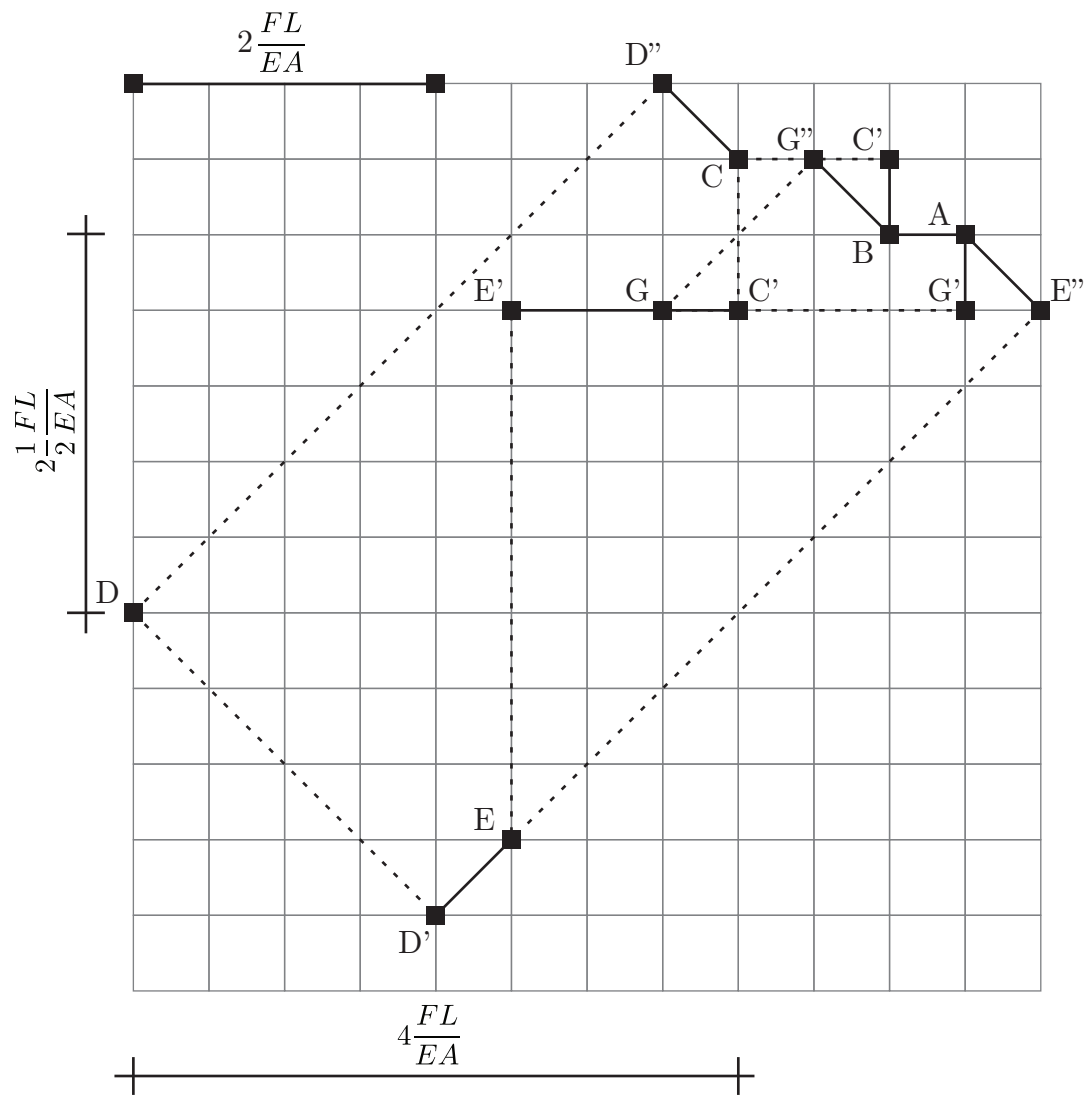


## Design of Structures – Exam Training – AM Solutions

Thom Bindels - t.h.w.bindels@student.tue.nl

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# 1 Assignment 2019-Feb Q1



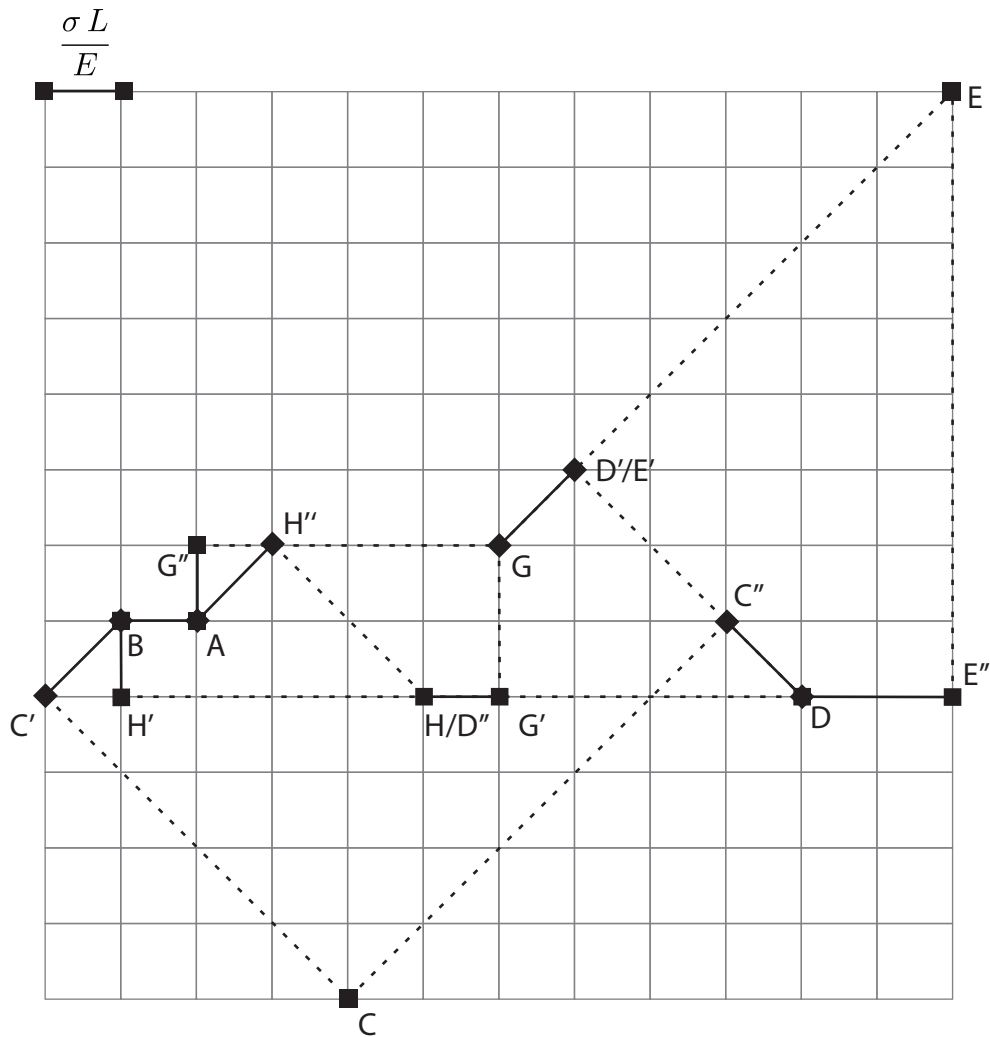
**MC1**

$$\begin{aligned}\Delta l_{\text{DC}} &= \frac{F_3 L_3}{E_3 A_3} \\ &= \frac{\frac{1}{2} F \sqrt{2} \sqrt{2} L}{EA \sqrt{2}} \\ &= \frac{1}{2} \frac{FL}{EA} \sqrt{2}\end{aligned}$$

**MC2**

$$\begin{aligned}\Delta l_{\text{AB}} &= \frac{F_5 L_5}{E_5 A_5} \\ &= \frac{-\frac{1}{2} FL}{EA} \\ &= -\frac{1}{2} \frac{FL}{EA}\end{aligned}$$

## 2 Assignment 2018-Jan Q2



Order:

A/B -> H -> G -> D -> C -> E

Important to realize is that the area  $A$  of each member is adjusted so that the stress is **always**  $\sigma$ . Therefore, in the expression for elongation  $\sigma = \frac{F}{A}$  can be substituted, giving the following equation for the elongation:

$$\Delta L = \frac{F L}{E A} = \frac{\sigma L}{E} \quad (1)$$

You can see from this equation that you only need to nature of the force/stress (tension or compression) and the length of the member; the other factors  $\sigma$  and  $E$  are constant for all members.

### MC9

$$\begin{aligned}\Delta l_{ED} &= \frac{F_3 L_3}{E_3 A_3} \\ &= \frac{-\sigma 2L}{E} \\ &= -2 \frac{\sigma L}{E}\end{aligned}$$

As the elongation is a minus (-) result, it is indeed shortening. In this case answer E is correct.

### MC10

$$\begin{aligned}\Delta l_{GD} &= \frac{F_9 L_9}{E_9 A_9} \\ &= \frac{\sigma \sqrt{2}L}{E}\end{aligned}$$

You would expect answer B to be correct. **However**, the question specifically asks for shortening of member GD, but in reality it is elongating due to the positive force in the member. Therefore, for the question to be correct (that asks for shortening), the answer should have been as a minus (-) result. (In other words, member GD is shortening with  $-\frac{\sigma \sqrt{2}L}{E}$  or elongating by  $\frac{\sigma \sqrt{2}L}{E}$ .)

## 3 Assignment 2018-Jan Q3

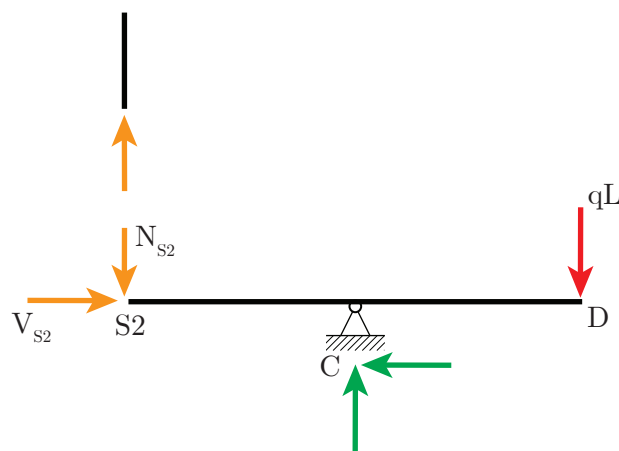
### MR14

The following answers are wrong for the given reasons:

- **A)** This is not true, segment S1-S2 causes a reaction force in the middle of beam A-B. Therefore, the displacement of A/B in the middle is also countered by that reaction force caused by S1-S2, resulting in less displacement.
- **E)** The standard beam equations never take care of the inclined beam effect by default. If a point rotates in the structure, you must be aware that the adjoining part of the beam starts to rotate as well, causing additional displacements.

### MC15

To find the axial force in member S1-S2 can be found by finding equilibrium in the free body diagram of the bottom segment:





Moment equilibrium about C gives:

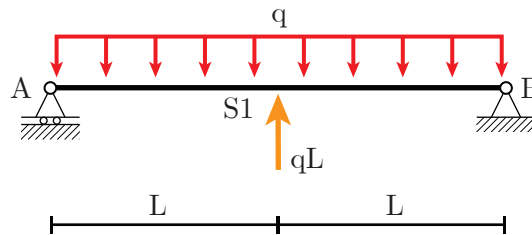
$$\sum M_C = 0 : \quad N_{S2} \cdot L - qL \cdot L = 0 \quad (2)$$

$$N_{S2} = qL \quad (3)$$

Beware that this means that the axial force **within** member S1-S2 is of compressive nature, which results in a correct answer of  $F_{S1-S2} = -qL$ .

### MC16

To find the displacement of S1, the following FBD can be constructed:



Use the following standard beam equations:

|  |   |
|--|---|
|  | $\varphi_A = \frac{FL^2}{16EI} \quad \zeta \varphi_B = \frac{FL^2}{16EI}$ $\downarrow w_{middle\_AB} = \frac{FL^3}{48EI}$             |
|  | $\varphi_A = \frac{qL^3}{24EI} \quad \zeta \varphi_B = \frac{qL^3}{24EI}$ $\downarrow w_{middle\_AB} = \frac{5}{384} \frac{qL^4}{EI}$ |

This results in

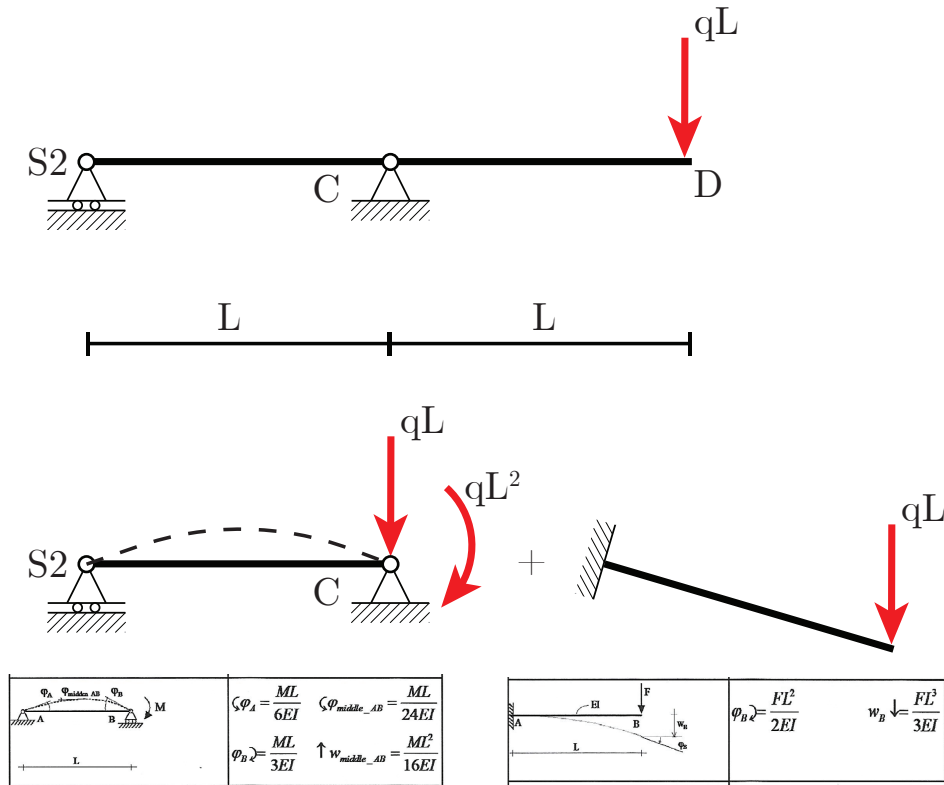
$$w_{S1} = \frac{5}{384} \frac{qL^4}{EI} - \frac{FL^3}{48EI} \quad (\text{standard beam form}) \quad (4)$$

$$w_{S1} = \frac{5}{384} \frac{q(2L)^4}{EI} - \frac{3qL(2L)^3}{48EI} \quad (5)$$

With the standard beam equations, always be aware of using the correct length! In the SBE, L is often the length of the beam, whereas many exercises have different lengths such as here  $2L$ .

### MC17

To find the displacement of D, the following FBD can be created with the corresponding SBE's:



First, the rotation in point C is calculated using the moment:

$$\varphi_C = \frac{ML}{3EI} \quad (6)$$

$$= \frac{qL^2 \cdot L}{3EI} \quad (7)$$

$$= \frac{qL^3}{3EI} \quad (8)$$

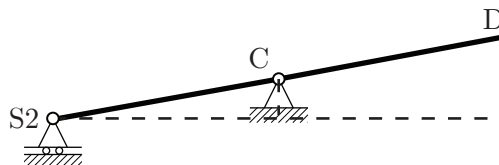
Next, the inclined beam effect due to this rotation and the additional bending due to the point load give the displacement of point D:

$$w_D = \frac{FL^3}{3EI} + \varphi \cdot L \quad (9)$$

$$= \frac{qL \cdot L^3}{3EI} + \frac{qL^3}{3EI} L \quad (10)$$

$$= \frac{2}{3} \frac{qL^4}{EI} \quad (11)$$

Be aware that this exercise can be finalized by one more step:

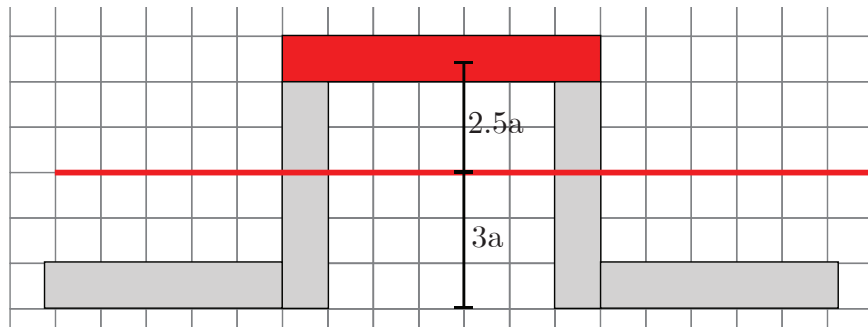


Due to the displacement of S2, the beam rotates about C, causing a vertical displacement of point D. In these cases, simply use the fact that the ratios of the sides must be equal.

## 4 Assignment 2018-Jan Q1

### MC4

The following situation is considered:



The second moment of area consists of two parts: 1) The part dependent on the shape of the block (for rectangles this is  $\frac{1}{12}bh^3$ ; and 2) The Steiner part which is the area multiplied by the distance of the shape center of gravity to the cross section neutral axis  $z_g$ . To find the second moment of area of the entire section you do this for all the “sub”-shapes you chose and sum their contributions.

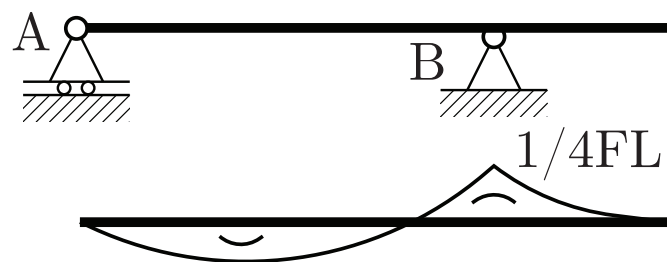
### MC7

The maximum bending stress can be calculated using

$$\sigma_b = \frac{Mz}{I} \quad (12)$$

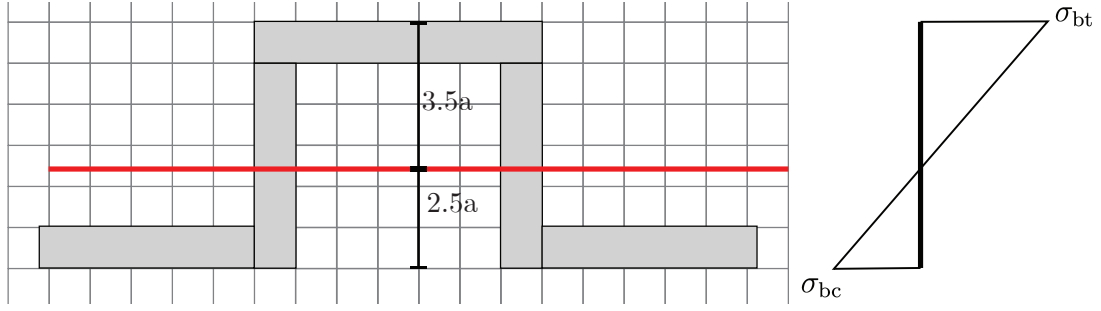
The largest bending stresses occur at the outermost fibers of the cross section. **Important:** If a cross section is not symmetric vertically, the neutral axis may be closer to one of the two ends. This results in two different maximum bending stresses (different for tension and compression side of the member). That is the case here as well.  $z$  is the distance from the neutral axis to the point where you want to calculate your bending stress, for maximum this were the outermost fibers: So from the neutral axis to the top and bottom of your section.

$M$  is the largest bending moment found in your entire structure. For this problem, it is shown in the following diagram:



Be very careful now with your reasoning! The maximum bending moment occurs at point B, but you can see that a so-called “hogging” bending moment occurs there: A negative bending moment. As you know, normally when you bend a beam downwards (sagging: positive bending), you find tension at the bottom side and compression at the top. However, when you deal with a negative bending moment such as in point B, this is reversed! The tensile zone is at the top and the compressed zone at the bottom.

This results in the bending moment diagram over the height of the cross section as follows:



With tension at the top and compression at the bottom. The maximum tensile stress is

$$\sigma_{bt} = \frac{\left(\frac{1}{4}FL\right) \cdot 3.5a}{\frac{2047}{24}a^4} = \frac{21}{2047} \frac{FL}{a^3} \quad (13)$$

Compressive maximum stress is calculated as

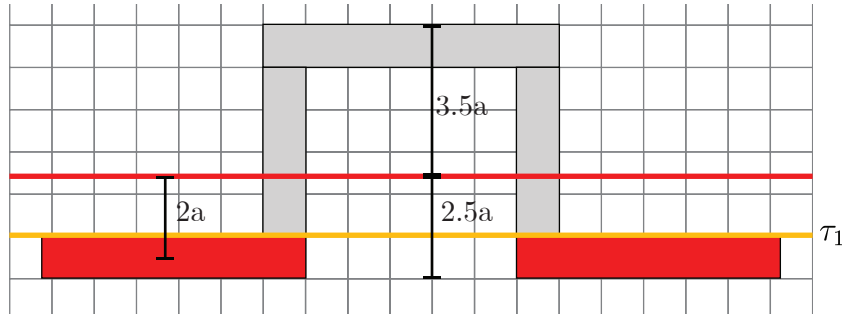
$$\sigma_{bc} = -\frac{\left(\frac{1}{4}FL\right) \cdot 2.5a}{\frac{2047}{24}a^4} = -\frac{15}{2047} \frac{FL}{a^3} \quad (14)$$

### MR8

The maximum shear stress occurs at the neutral axis, and is zero at the outer fibers. It can be calculated using

$$\tau = \frac{VS}{bI} \quad (15)$$

The neutral axis is at  $z_g = 2.5a$ , which is  $\tau_2$ . Therefore,  $\tau_2$  must be the maximum shear stress as it is the location of the neutral axis. The shear stress at  $\tau_1$  is calculated using:



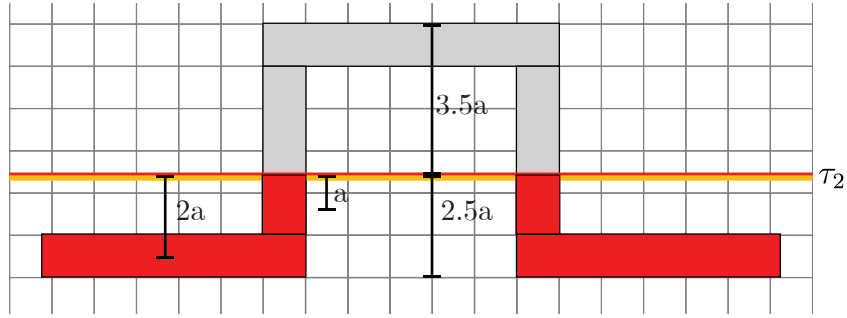
The first moment of area  $S$  is calculated with respect to one side of the section. In this case the bottom side is chosen as there are fewer elements there. This results in:

$$S = 2(6.25a \cdot a \cdot 2a) = 25a^3 \quad (16)$$

This gives a shear stress of ( $b$  is the total length where the top and bottom part of the section touches)

$$\tau_1 = \frac{VS}{bI} = \frac{F \cdot 25a^3}{2a \cdot \frac{2047}{24}a^4} = 0.1466 \frac{F}{a^2} \quad (17)$$

The shear stress at  $\tau_2$  is calculated using:



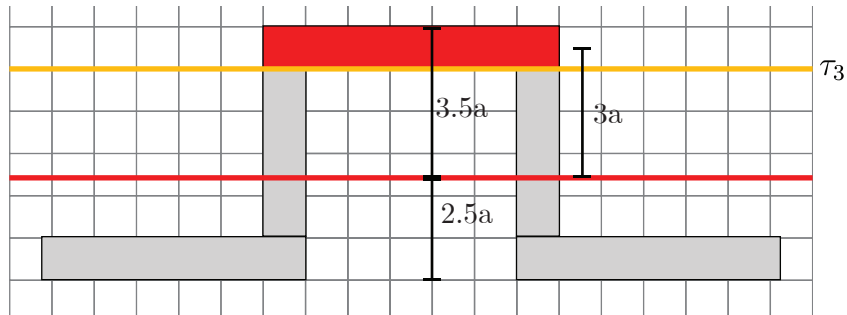
The first moment of area  $S$  is:

$$S = 2(6.25a \cdot a \cdot 2a + a \cdot 1.5a \cdot a) = 28a^3 \quad (18)$$

This gives a shear stress of

$$\tau_2 = \frac{VS}{bI} = \frac{F \cdot 28a^3}{2a \cdot \frac{2047}{24}a^4} = 0.1641 \frac{F}{a^2} \quad (19)$$

The shear stress at  $\tau_3$  is calculated using:



The first moment of area  $S$  is:

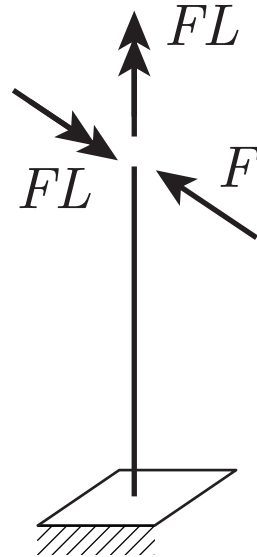
$$S = 7a \cdot a \cdot 3a = 21a^3 \quad (20)$$

This gives a shear stress of

$$\tau_3 = \frac{VS}{bI} = \frac{F \cdot 21a^3}{2a \cdot \frac{2047}{24}a^4} = 0.1231 \frac{F}{a^2} \quad (21)$$

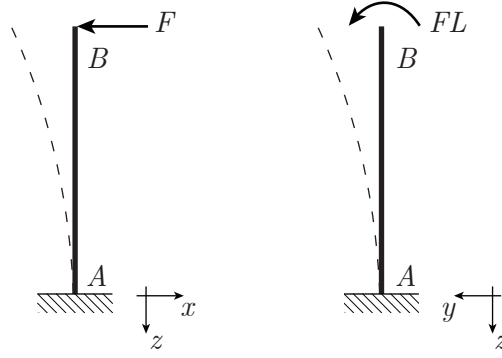
## 5 Assignment 2019-Feb Q2

Simplify to the figure below, you may ignore the vertical normal force as normal deflections can be ignored (as they are relatively small compared to deflections due to bending).



MC6

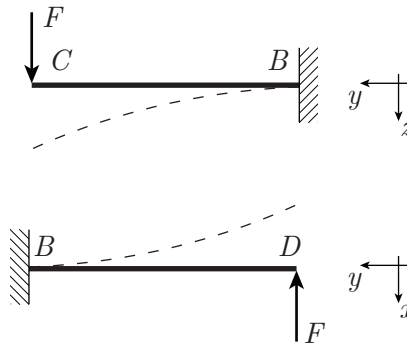
- **A)** Incorrect, part AB will have some rotation as well as displacements due to bending. This causes point B to both move and rotate. A hinge cannot capture this behavior, and it should be fictitiously clamped at point B instead.
- **B)** Incorrect, the force downwards at point C can cause part CBD to tilt. This causes a vertical displacement at point D as well.
- **C)** Incorrect (albeit not entirely sure), normally one can work their way back using an unknown angle. However, as there are external forces on the location to be clamped, we are not entirely sure whether this method can be applied. We believe it is most likely not possible.
- **D)** Incorrect, this is not likely. For example, the normal force acting on BA is  $F$ . This will cause a shortening of  $\frac{FL}{EA}$  of part BA. This is very little in comparison to the displacements caused by for example bending (which is a factor  $L^3$  compared to a factor  $L!$ ).
- **E)** Incorrect, the shear forces have a different orientation so their influence on the member is different.
- **F)** Correct, the only external forces act as shear loads on part CBD.



### MC7

Split in the two main planes  $xz$  and  $yz$ , do not forget the torsion component. If the directions are unclear, check the AM reader at page 177. In general, always use the smallest angle between two axes! From one to the other axis is then considered positive, as you indicate with your arrow.

$$\begin{aligned}\phi_{Bx,y \rightarrow z} &= \frac{FL^2}{EI} \\ \phi_{By,z \rightarrow x} &= \frac{FL^2}{2EI} \\ \phi_{Bz,y \rightarrow x} &= \frac{FL^2}{GI_P} \\ w_{Bx} &= -\frac{FL^3}{3EI} \\ w_{By} &= \frac{FL^3}{2EI} \\ w_{Bz} &= 0\end{aligned}$$



### MC8

Once you start fictitiously clamping your structure, do not forget the initial displacements of that point (in our case B as found before); as well as any effects due to the wagging tail/inclined beam effect.

$$\begin{aligned}\phi_{Cx,y \rightarrow z} &= \phi_{Bx,y \rightarrow z} + \frac{FL^2}{2EI} \\ \phi_{Cy,z \rightarrow x} &= \phi_{By,z \rightarrow x} \\ \phi_{Cz,y \rightarrow x} &= \phi_{Bz,y \rightarrow x} \\ w_{Cx} &= w_{Bx} + \phi_{Bz,y \rightarrow x}L \\ w_{Cy} &= w_{By} \\ w_{Cz} &= w_{Bz} + \phi_{Bx,y \rightarrow z}L + \frac{FL^3}{3EI}\end{aligned}$$

**MC9**

$$\phi_{Dx,y->z} = \phi_{Bx,y->z}$$

$$\phi_{Dy,z->x} = \phi_{By,z->x}$$

$$\phi_{Dz,y->x} = \phi_{Bz,y->x} + \frac{FL^2}{2EI}$$

$$w_{Dx} = w_{Bx} - \phi_{Bz,y->x}L - \frac{FL^3}{3EI}$$

$$w_{Dy} = w_{By}$$

$$w_{Dz} = w_{Bz} - \phi_{Bx,y->z}L$$

**MC10**

$$w_{Cx} = w_{Bx} + \phi_{Bz,y->x}L = -\frac{FL^3}{3EI} + \frac{FL^3}{GI_P}$$

$$w_{Cy} = w_{By} = \frac{FL^3}{2EI}$$

$$w_{Cz} = w_{Bz} + \phi_{Bx,y->z}L + \frac{FL^3}{3EI} = \frac{FL^3}{EI} + \frac{FL^3}{3EI} = \frac{4FL^3}{3EI}$$

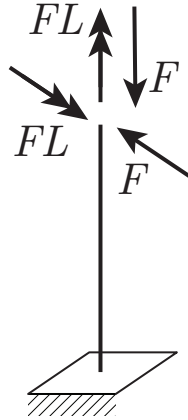


$$\begin{aligned}
\phi_{Bx,y \rightarrow z} &= \frac{FL^2}{EI} \\
\phi_{By,z \rightarrow x} &= \frac{FL^2}{2EI} \\
\phi_{Bz,y \rightarrow x} &= \frac{FL^2}{GI_P} \\
w_{Bx} &= -\frac{FL^3}{3EI} \\
w_{By} &= \frac{FL^3}{2EI} \\
w_{Bz} &= 0
\end{aligned}$$

MC11

$$\begin{aligned}
w_{Dx} &= w_{Bx} - \phi_{Bz,y \rightarrow x}L - \frac{FL^3}{3EI} = -\frac{FL^3}{3EI} - \frac{FL^3}{GI_P} - \frac{FL^3}{3EI} = -\frac{2FL^3}{3EI} - \frac{FL^3}{GI_P} \\
w_{Dy} &= w_{By} = \frac{FL^3}{2EI} \\
w_{Dz} &= w_{Bz} - \phi_{Bx,y \rightarrow z}L = -\frac{FL^3}{EI}
\end{aligned}$$

MC12



- **A)** Correct, there is a constant normal force along the beam due to  $F$ . This creates a constant normal stress over the length AB. Additionally, a non-constant bending moment will be present over the length of the beam. At the location where this is largest, the normal stress due to bending can be calculated. Then, the maximum normal stress along AB is at that location and the summation of the axial stress component and the bending stress component.
- **B)** Correct, it is subjected to bending about the x-axis, a shear force  $F$  causing bending about the y-axis, torsion about the z-axis and a normal force.
- **C)** Incorrect

$$\begin{aligned}
M_{Ed} &= 100 \cdot 2 = 200 \text{ kNm} \\
I &= \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(180^4 - 160^4) = 6162500\pi \text{ mm}^4 \\
\sigma_b &= \frac{M_{Ed}z}{I} = \frac{200 \cdot 10^6 \cdot 90}{6162500\pi} = 930 \text{ MPa}
\end{aligned}$$

- **D)** Correct, (assuming the length is still 2 m), see previous answer. The stresses due to bending are already exceeding the material strength. It becomes even worse as there will be an additional axial force of 100 kN, that increases the compressive stresses.
- **E)** Incorrect, the bending about x-axis is indeed constant as there is no additional shear force in the direction. However, the torsional moment is also constant over AB.
- **F)** Incorrect, torsional bending moments cause shear stresses in the material. The bending moment creates normal stresses. Therefore, the torsional bending moment reduces the shear capacity of AB and not the bending moment/normal stress capacity.