

# Statics of Structures – Exam Training

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## 1 Diagram Procedure

1. Draw the axes that are considered positive in your calculations.<sup>a</sup>



Figure 1: A possible choice for positive axes.

2. Draw and name all the (still unknown) reaction forces; make an assumption for their direction.<sup>b</sup>

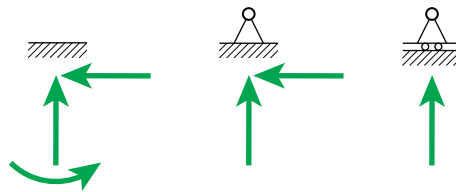


Figure 2: Fixed, hinged and roller reaction forces.

3. Count the number of reaction forces. Are there more than 3? Then you need additional equilibrium equations to the three global equilibrium equations. Keep this in mind.
4. Draw the free body diagrams needed to solve for the reaction forces. This at least includes one of the entire structure. If there were more than 3 unknown reaction forces, additional free body diagrams must be constructed that provided additional equilibrium equations (these are the hinges, we know a bending moment must be zero here!). Where a hinge is located, make a cut through the hinge, and choose the either the left or right side of the cut to evaluate. **Do not forget** to draw the normal and shear internal forces that can occur at the cut of the hinge.<sup>c</sup>

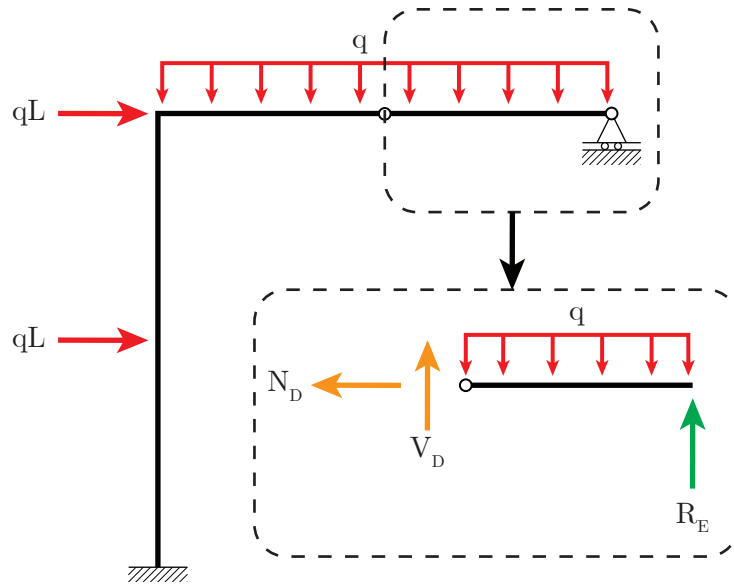


Figure 3: Example of a structure with more than 3 unknowns, but also a hinge in the structure. The hinge by definition cannot withstand any rotation. Therefore, the bending moment **must** be zero. We can use this information to obtain an additional equilibrium equation.

5. Using all the free body diagrams, write all the equilibrium equations down:
  - $\sum F_h = 0$ : All horizontal forces in the entire structure must sum to zero.
  - $\sum F_v = 0$ : All vertical forces in the entire structure must sum to zero.
  - $\sum M = 0$ : All bending moments about a certain point must sum to zero.<sup>d</sup>
  - $\sum M_{S_i} = 0$ : In case that there are more than 3 unknowns, moment equilibrium about hinges may be required. Make a cut at the hinge and perform moment equilibrium to one side of the cut.
6. Solve the equations to find all the reaction forces. The first part is now finished. Next are the diagrams.
7. Start by separating any members that are at an angle to each other. At the ends where they are disconnected, the correct internal force couples must be placed to maintain internal equilibrium.

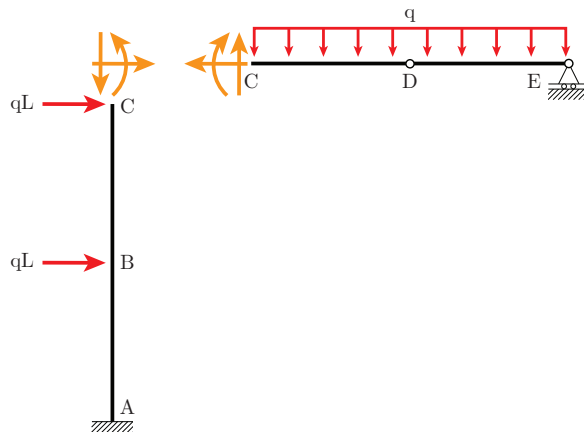


Figure 4: Example of a structure and how it can be split in linear segments; where split the correct internal force pairs are placed.

8. Now start with the normal force diagram. Look at each segment and see if there are parts that are subjected to axial forces. If there are none, indicate this in your diagram with a “zero” dot on that member. Moreover, make sure to use the correct symbols to indicate either tension or compression!
9. Next, do the same but for the shear (transverse) forces. To construct this diagram, start at the **left** side of a member. Now simply follow the forces and sketch along. If there is a force pointing upwards at the left start, the diagram starts with this magnitude in this direction, go along the member and once you encounter a shear force you simply add/subtract it from your diagram at that location.

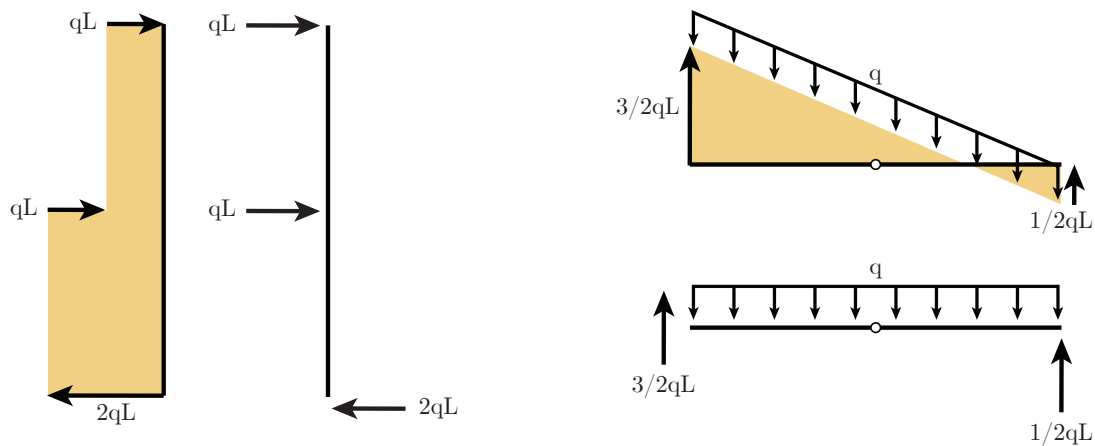


Figure 5: Construction of shear force diagrams from the free body diagrams of member segments.

10. This only leaves the bending moment diagram to finish. This one can be constructed fully from the shear force diagrams, which is due to the relationship  $M = \int V dx$ . What this equation basically says: “The area under your shear force diagram, is the change in bending moment along that segment”. The following steps must be taken:
  - (a) Pick a starting point where the bending moment is known or zero (either a fixed connection or a hinge).
  - (b) Calculate the area of the shear force diagram from that point until it changes.
  - (c) Use the symbol of the shear force to determine whether the bending moment decreases or increases.
  - (d) Draw the bending moment diagram from the starting point, to the new point (the old point minus/plus the calculated area under the shear force diagram). Take into account that a constant shear force becomes a linear bending moment and a linear shear force becomes a quadratic bending moment.
  - (e) Repeat this process until the diagram is complete.

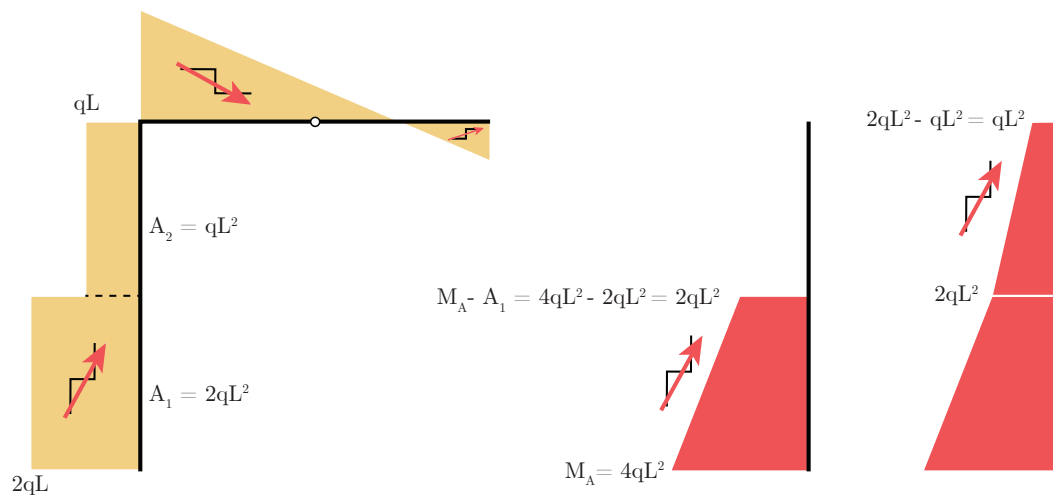


Figure 6: Bending moment procedure.

<sup>a</sup>: It simply indicates what direction on your paper you consider positive and which one negative. This may be any choice, but you need to be consistent throughout your exercises with this choice. This choice is different from the chosen conventions for positive/negative (internal) forces that you are taught as well. For example, if you make a right-hand cut, the internal forces are oriented in a certain manner.

<sup>b</sup>: Only with engineering insight you can know beforehand what the actual direction of a reaction force will be. Try to guess them right but do not worry if you are not sure. Once you calculated your reaction forces, any wrong directions will reveal themselves simply as a negative force. If you calculated reaction forces that are negative, you can flip their direction and use the same magnitude (except the minus).

<sup>c</sup>: When making a cut through a hinge, you should choose the side of the cut that has the least forces/complexities. This makes your calculations easiest. You **cannot** use moment equilibrium about both the left and right side of the cut! You can only use it effectively for one side, as you are basically considering the same equilibrium condition. Doing it for both sides will not give you any new information and you will lose time.

<sup>d</sup>: You can choose **any** point about which you take moment equilibrium; it may even be “floating” in space and not directly on the structure. Try to choose this point strategically: Choose a point where at least some forces intersect (for example a support). This is because when forces intersect, they do not have a lever arm, and thereby do not contribute to the bending moment about that specific point. This reduces the complexity of the equation.



## 2 Elaborations

### 2.1 Exam 2019 – Mechanics Question 1

The first step is to find the unknown reaction forces in Figure 7. There is a fixed support at A and a roller at E, this gives a total of 4 unknown reaction forces, more than 3. Therefore, we need to utilize the hinge in point D as well (where we know the bending moment must be zero).

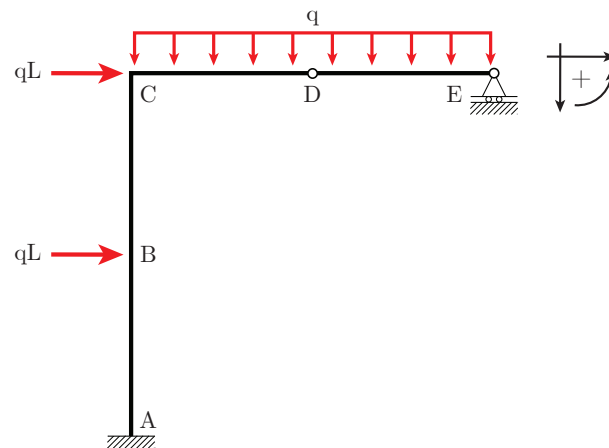


Figure 7: Mechanical scheme.

For the free body diagram at the hinge, it is chosen to do this with respect to the left of the structure. This side clearly has the fewest forces/moments and makes calculations easier. Do not forget the internal normal and shear force at the point D. The internal moment at point D is left out, as we know this is a hinge and this moment must be equal to zero. The free body diagrams of the whole structure and the cut section at D are shown in Figure 8.

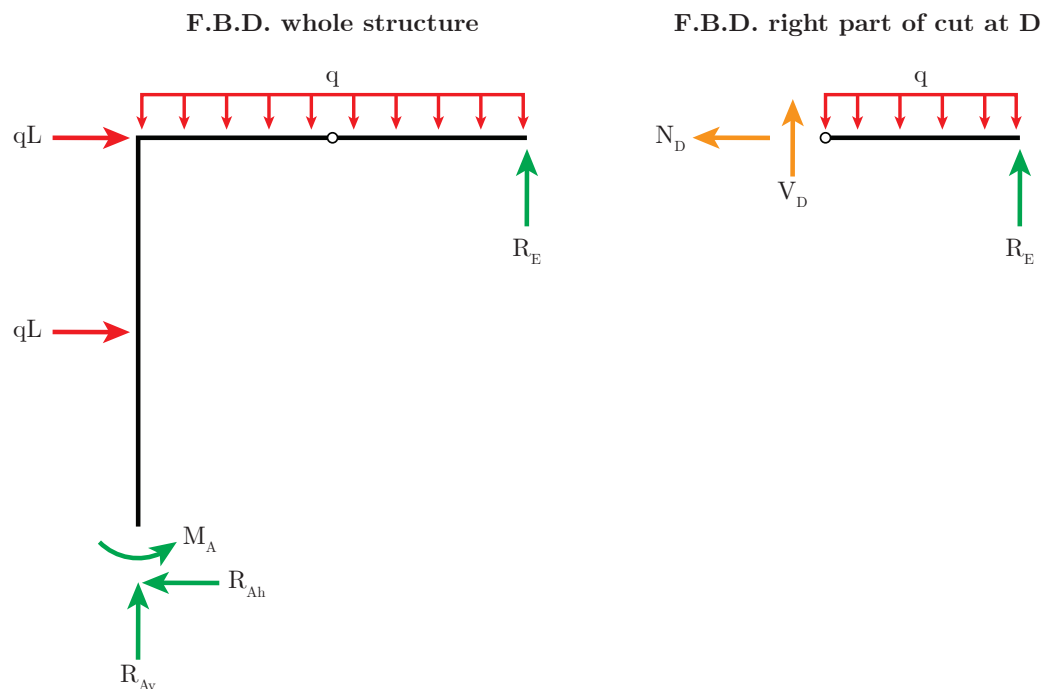


Figure 8: Free body diagrams to solve the reaction forces.

Next, the four equilibrium conditions to solve for the reaction forces are defined. First start with global equilibrium concerning the whole structure (moment equilibrium is taken about point A):

$$\sum F_h = 0 : \quad qL + qL - R_{Ah} = 0 \quad (1)$$

$$R_{Ah} = 2qL \quad (2)$$

$$\sum F_v = 0 : \quad q \cdot 2L - R_{Av} - R_E = 0 \quad (3)$$

$$R_{Av} + R_E = 2qL \quad (4)$$

$$\sum M_A = 0 : \quad M_A - qL \cdot L - qL \cdot 2L - q \cdot 2L \cdot L + R_E \cdot 2L = 0 \quad (5)$$

$$M_A + R_E \cdot 2L = qL^2 + 2qL^2 + 2qL^2 \quad (6)$$

$$M_A + R_E \cdot 2L = 5qL^2 \quad (7)$$

Next, moment equilibrium at point D is defined as:

$$\sum M_D = 0 : \quad R_E \cdot L - q \cdot L \cdot \frac{L}{2} = 0 \quad (8)$$

$$R_E = \frac{1}{2}qL \quad (9)$$

Substitution of Equation (9) in Equations (4) and (7) gives:

$$R_{Av} = \frac{3}{2}qL \quad (10)$$

$$M_A = 4qL^2 \quad (11)$$

Now the reaction forces are known, the diagrams must be constructed. First, the structure is split into linear parts so that the diagrams can be constructed easily. In this case, that means simply a vertical and horizontal segment (it is cut in point C). First, the top segment is assessed, and the correct internal forces are placed at the cut in location C. This is shown in Figure 9.

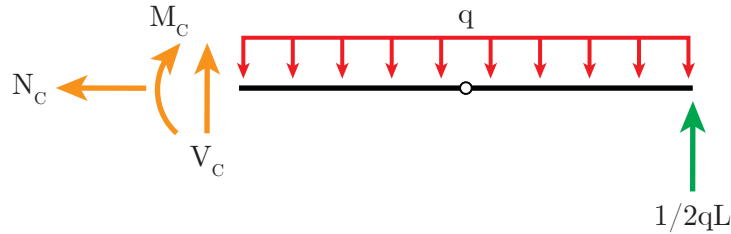


Figure 9: Top segment of the structure.

Equilibrium must also hold for this separated segment. Using the three main equilibrium equations the internal forces are calculated as:

$$\sum F_h = 0 : \quad N_C = 0 \quad (12)$$

$$\sum F_v = 0 : \quad q \cdot 2L - V_C - \frac{1}{2}qL = 0 \quad (13)$$

$$V_C = \frac{3}{2}qL \quad (14)$$

$$\sum M_A = 0 : \quad -M_C - q \cdot 2L \cdot L + \frac{1}{2}qL \cdot 2L = 0 \quad (15)$$

$$-M_C - 2qL^2 + qL^2 = 0 \quad (16)$$

$$M_C = -qL^2 \quad (17)$$

These internal forces are now shown in Figure 10. Moreover, as an internal cut must be in equilibrium, these forces but in opposite directions act at the top of the vertical segment as well.

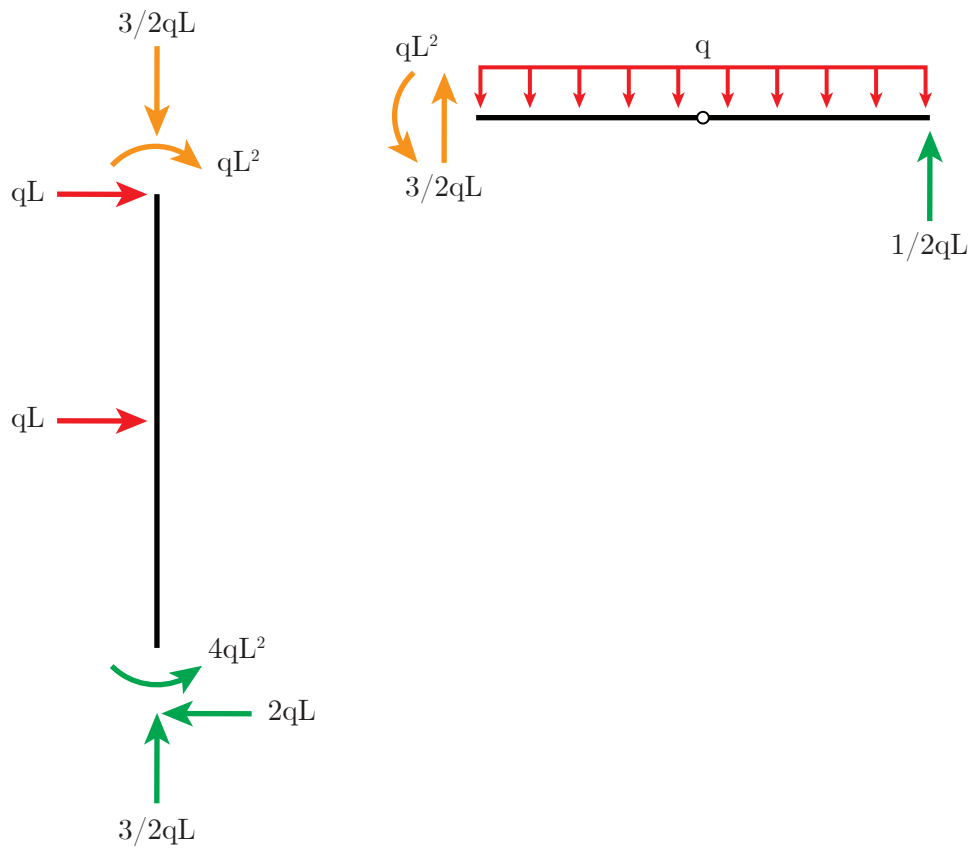


Figure 10: Separated segments and their internal forces of the entire structure.

Now, everything required to construct the diagrams is prepared. First, the normal force diagram is constructed. In this situation, it is relatively simple. The top segment does not have any axial forces, so the normal force diagram is zero. The vertical bar is compressed by a force of  $\frac{3}{2}qL$  at both ends of the bar, therefore the entire bar is subjected to this force in compression. This results in the normal force diagram shown in Figure 11.

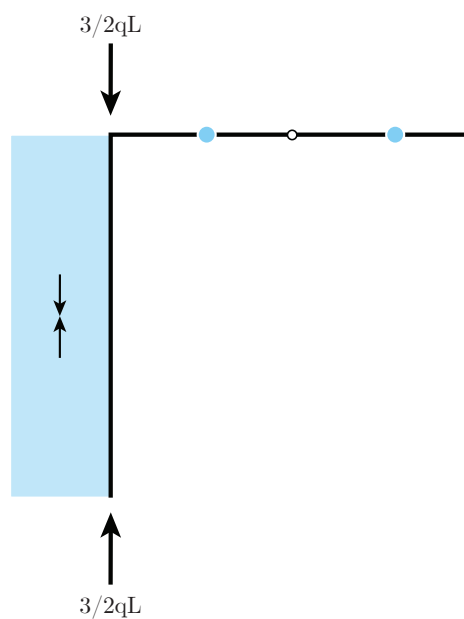


Figure 11: Normal force diagram.

The shear force diagram can be constructed by following the shear forces (and changes thereof) along the length of a member, starting at the left and going right! This results in the diagram shown in Figure 12.

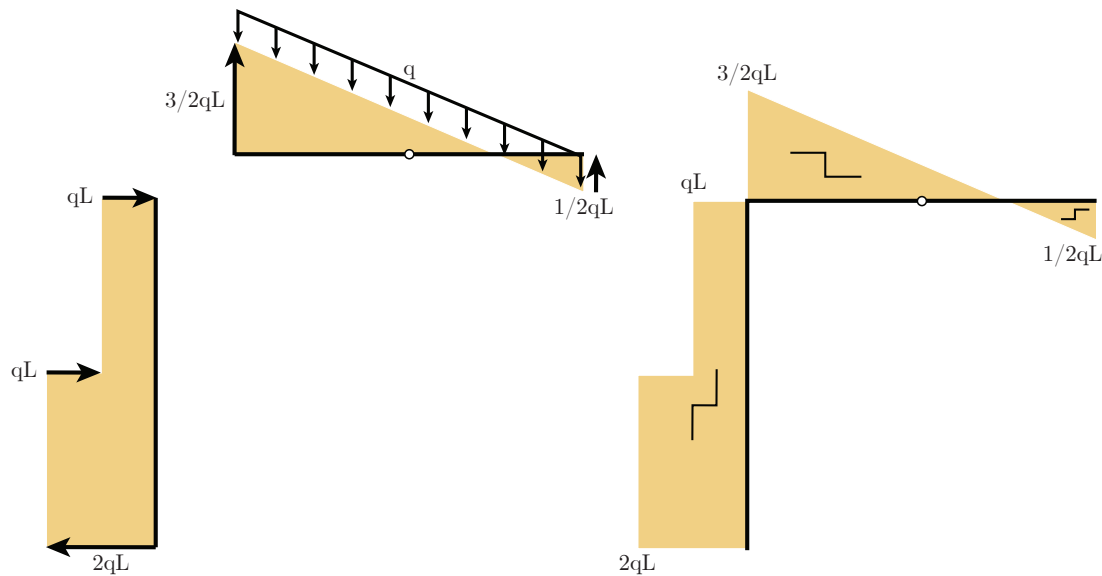


Figure 12: Shear force diagram.

The bending moment can be calculated using the shear force diagram. First, a starting point must be chosen. It is easiest to work again from left to right, therefore it is logical to start in point A. Important to remember, this point is fixed and not a hinge! Therefore, the bending moment does not start at zero there. Actually, the bending moment there is equal to the previously calculated value of  $M_A$ .

Next, the area below the shear force diagram is required. The area of a section indicates how much the bending moment increases or decreases. For example, area  $A_1 = 2qL \cdot L = 2qL^2$  is how much the bending moment changes from point A until point B. Next, it must be decided whether this is an increase or decrease in bending moment. For this, the shear force symbols can be used. Simply draw an arrow in the direction you are working through the symbol. This indicates whether it would increase or decrease. Keep on going through the structure like this. Be aware the moments can be transferred around corners, such as point C. Therefore, the moment at the top of the vertical member and the left of the horizontal member is equal.

For the horizontal segment the shear force is linear and not constant. As a result, the bending moment diagram has a quadratic shape (and for a constant shear force the bending moment diagram is linear). You can see that the bending moment diagram passes exactly through the hinge in D as well. This is also a good way to check yourselves, as you know that the bending moment in a hinge must be zero! Lastly, the bending moment changes its direction for the last  $\frac{1}{4}L$ , returning it to zero at the roller (which it must as there is no moment in the roller)!

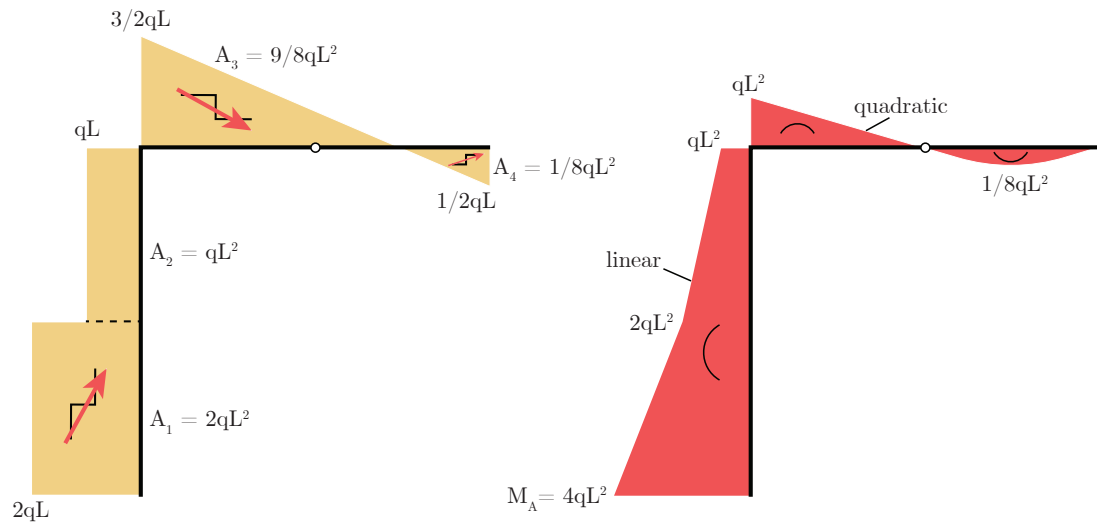


Figure 13: Bending moment diagram.

## 2.2 Exam 2019 – Mechanics Question 3

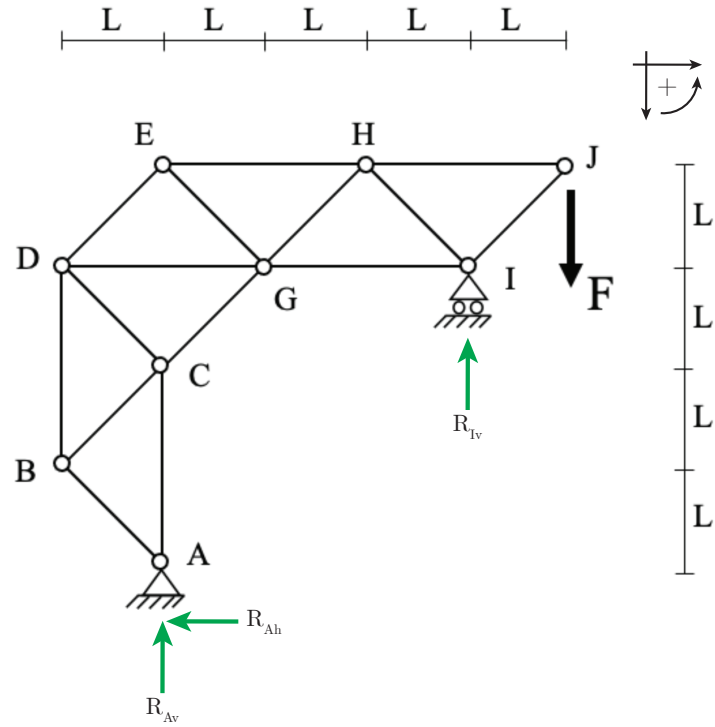


Figure 14: Mechanical scheme and reaction forces.

$$\sum F_h = 0 : \quad R_{Ah} = 0 \quad (18)$$

$$\sum M_A = 0 : \quad R_{Iv} \cdot 3L - F \cdot 4L = 0 \quad (19)$$

$$R_{Iv} = \frac{4}{3}F \quad (20)$$

$$\sum F_v = 0 : \quad F - R_{Av} - R_{Iv} = 0 \quad (21)$$

$$R_{Av} = -\frac{1}{3}F \quad (22)$$

$$(23)$$

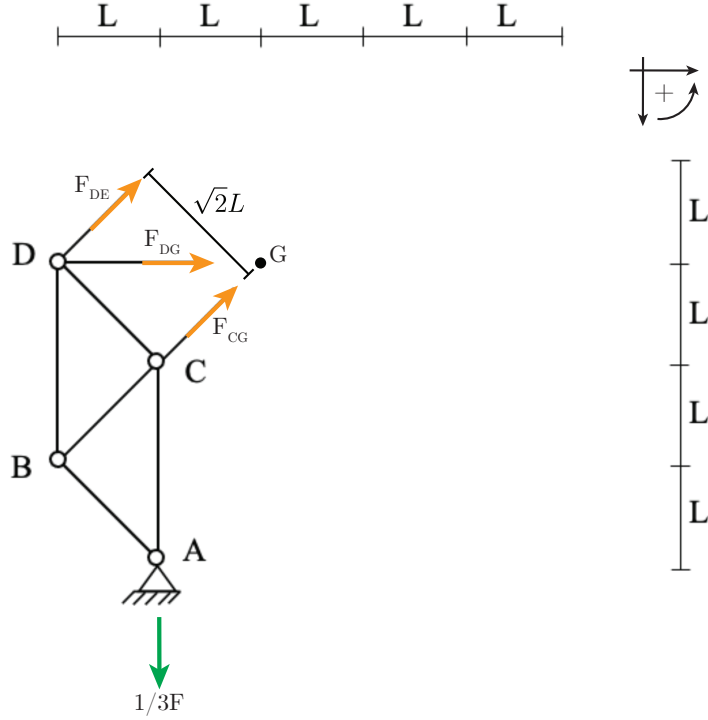


Figure 15: Free body diagram of the section.

To find the internal forces, use is made of the fact that at each node (hinge) the bending moment must be zero. First, moment equilibrium about point G is taken. This is chosen as here two of the unknowns intersect; meaning they do not create a moment about point G as their lever arm is zero. This way we can solve for  $F_{DE}$ :

$$\sum M_G = 0 : \quad \frac{1}{3}F \cdot L - F_{DE} \cdot \sqrt{2}L = 0 \quad (24)$$

$$F_{DE} = \frac{1}{3\sqrt{2}}F \quad (25)$$

Next, the same trick is applied to solve for  $F_{CG}$  by finding moment equilibrium about point D:

$$\sum M_D = 0 : \quad -\frac{1}{3}F \cdot L + F_{CG} \cdot \sqrt{2}L = 0 \quad (26)$$

$$F_{CG} = \frac{1}{3\sqrt{2}}F \quad (27)$$

Last, point C is chosen to find  $F_{DG}$ :

$$\sum M_C = 0 : \quad -F_{DG} \cdot L - F_{DE} \cdot \sqrt{2}L = 0 \quad (28)$$

$$-F_{DG} \cdot L - \frac{1}{3\sqrt{2}}F \cdot \sqrt{2}L = 0 \quad (29)$$

$$F_{DG} = -\frac{1}{3}F \quad (30)$$

Next, the method of joints is applied to find additional member forces. This is done in point D, as shown in Figure 16. Horizontal and vertical equilibrium equations can be used in these situations, resulting in:

$$\sum F_h = 0 : \quad F_{DG} + \frac{\sqrt{2}}{2}F_{DE} + \frac{\sqrt{2}}{2}F_{DC} = 0 \quad (31)$$

$$\sum F_v = 0 : \quad -\frac{\sqrt{2}}{2}F_{DE} + \frac{\sqrt{2}}{2}F_{DC} + F_{DB} = 0 \quad (32)$$

Substituting  $F_{DE}$  and  $F_{DG}$  gives:

$$\sum F_h = 0 : -\frac{1}{3}F + \frac{\sqrt{2}}{2} \frac{1}{3\sqrt{2}}F + \frac{\sqrt{2}}{2}F_{DC} = 0 \quad (33)$$

$$-\frac{1}{6}F + \frac{\sqrt{2}}{2}F_{DC} = 0 \quad (34)$$

$$F_{DC} = \frac{1}{3\sqrt{2}}F \quad (35)$$

$$\sum F_v = 0 : -\frac{\sqrt{2}}{2} \frac{1}{3\sqrt{2}}F + \frac{\sqrt{2}}{2}F_{DC} + F_{DB} = 0 \quad (36)$$

$$-\frac{\sqrt{2}}{2} \frac{1}{3\sqrt{2}}F + \frac{\sqrt{2}}{2} \frac{1}{3\sqrt{2}}F + F_{DB} = 0 \quad (37)$$

$$F_{DB} = 0 \quad (38)$$

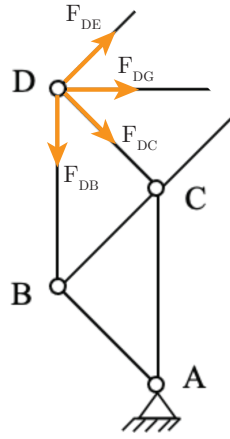


Figure 16: Method of joints at point D.

Either method can be used to find the forces, the final results are shown in Figure 17.

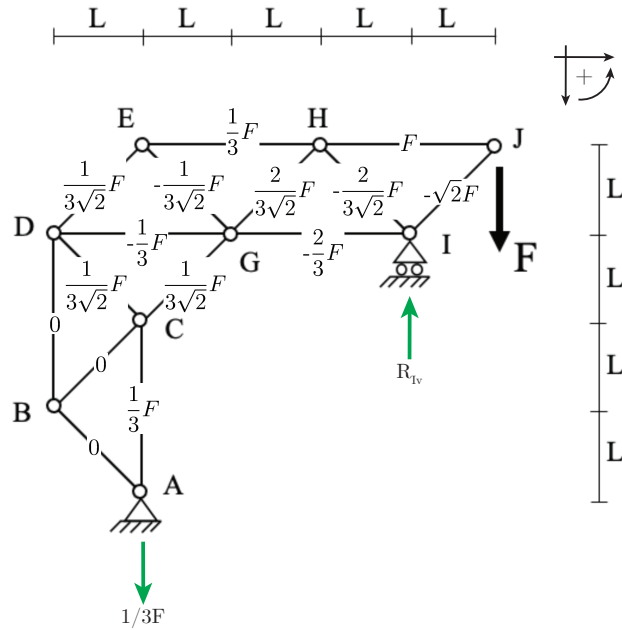


Figure 17: All truss member forces.



Last, when making these diagrams for the method of joints, **do not forget the internal force pairs and the orientation of the “cut”!** For example, as shown in Figure 18, we already knew the force  $F_{DE}$  from the method of sections. In that method, we assumed a direction for the internal force where we made the cut; this was a right hand cut. However, when we move to point E (which is on the right of the cut), we cannot simply “move” our original arrow to that spot. This is because at that side of the cut it is actually the left hand part of the cut, meaning the force must be flipped!

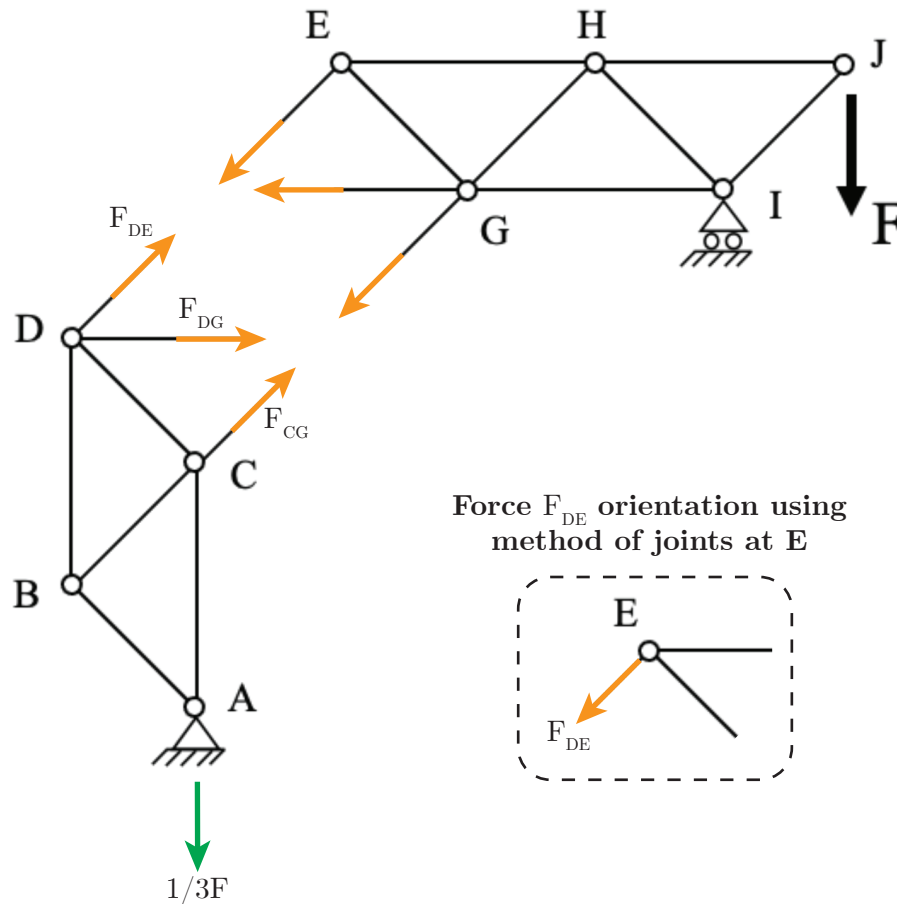


Figure 18: When constructing the method of joints forces, be very precise in the orientation of the arrows!